The Robinson-Schensted-Knuth correspondence

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Abstract

The Robinson–Schensted–Knuth (RSK) correspondence is among the most important bijections in algebraic and enumerative combinatorics. It generalizes a bijection between two ubiquitous combinatorial objects, permutations and (pairs of) Young tableaux. We will take a tour of the RSK correspondence from the perspective of Fomin's beautiful "growth diagrams". This approach to RSK leads more easily to many important properties and generalizations.

Contents

Question

Why beautiful bijections?

Permutations

Definition

A **permutation** of n is a rearrangement of the numbers $1, 2, \ldots, n$

Example

The six permutations of 3 are 123, 132, 213, 231, 312, and 321.

Fact

There are $n!$ permutations of n.

The symmetric group

Definition

Permutations form the **symmetric group** $\mathfrak{S}_n$ under composition (as rearrangements).

The study of $\mathfrak{S}_n$ leads to its irreducible representations, which are indexed by *partitions* and described in terms of *Young tableaux*.

Ferrers diagrams

Definition

A **partition** λ of n is a nonincreasing list $(\lambda_1, \lambda_2, \ldots, \lambda_d)$ of positive integers whose sum is n. We write $\lambda \vdash n$. The **Ferrers diagram** of a λ is a set boxes drawn justfied into a corner whose row lengths are $\lambda_1, \lambda_2, \ldots, \lambda_d$.

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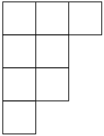
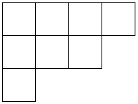
Example

For example, $\lambda = (3, 2, 2, 1) \vdash 8$ has Ferrers diagram



and the **transpose** is $(3, 2, 2, 1)^{\mathsf{T}} = (4, 3, 1)$



Standard Young tableaux

Definition

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For example:

1	3	4
2	6	
5	7	
8		

For non-example:

1	6	4
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7	5	
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Standard Young tableaux

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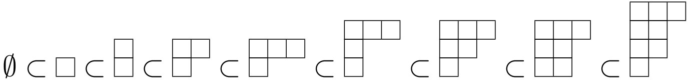
We can also think of this as a sequence of steps, each adding one box. For the example above,



Counting Young tableaux

Theorem (Frame–Robinson–Thrall)

The number of standard Young tableaux f_λ is given by the **hook-length** *formula*

$$f_\lambda = \frac{n!}{\prod_{c \in \lambda} h_c}$$

where h_c is the size of the hook at a cell c (includes c and all cells below or to the right).

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Example

Labeling the hook sizes of $(4, 2, 1)$, we obtain

6	4	2	1
3	1		
1			

$$f_{(4,2,1)} = \frac{7!}{6 \cdot 4 \cdot 2 \cdot 1 \cdot 3 \cdot 1 \cdot 1} = 35.$$

A surprising connection between $n!$ and f_λ

$$\sum_{\lambda \vdash n} f_\lambda^2 = n!$$

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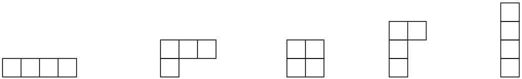
For $n = 4$,



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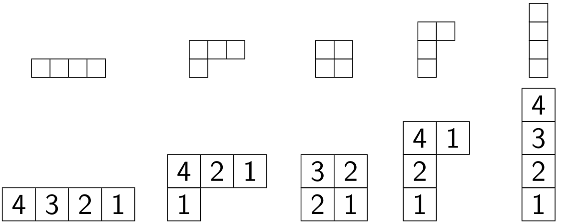


$\frac{4!}{4 \cdot 3 \cdot 2 \cdot 1}$ $\quad \frac{4!}{4 \cdot 2 \cdot 1 \cdot 1}$ $\quad \frac{4!}{3 \cdot 2 \cdot 2 \cdot 1}$ $\quad \frac{4!}{4 \cdot 2 \cdot 1 \cdot 1}$ $\quad \frac{4!}{4 \cdot 3 \cdot 2 \cdot 1}$

$$1^2 + 3^2 + 2^2 + 3^2 + 1^2 = 24 = 4!$$

A surprising connection between $n!$ and f_λ

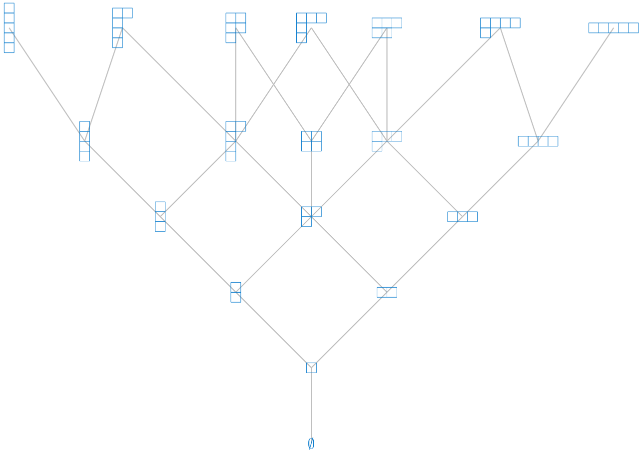
$$\sum_{\lambda \vdash n} f_\lambda^2 = n!$$

This might remind you of the classic identity and undergraduate combinatorial proof exercise.

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$

Young's lattice

$$\sum_{\lambda \vdash n} {f_\lambda}^2 = n!$$



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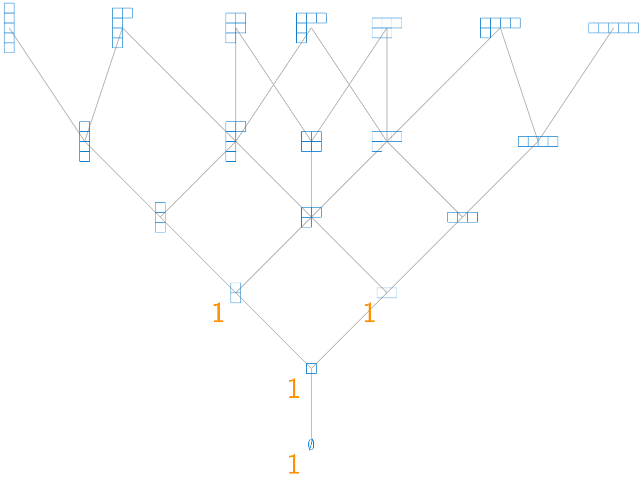

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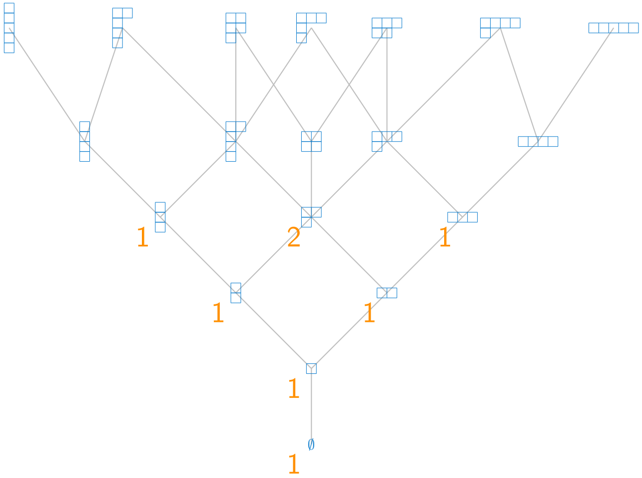

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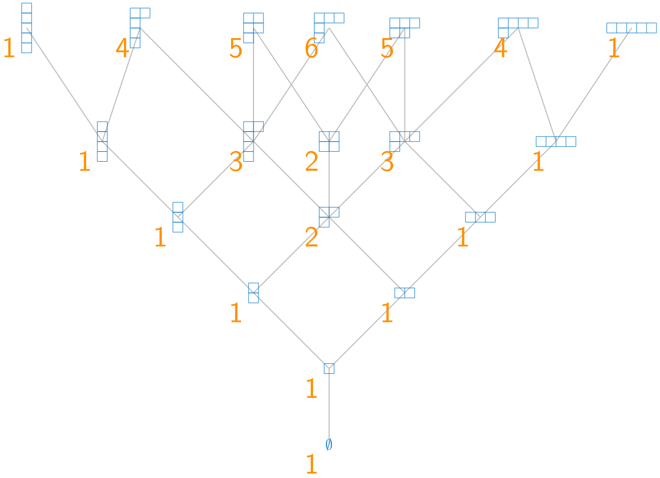

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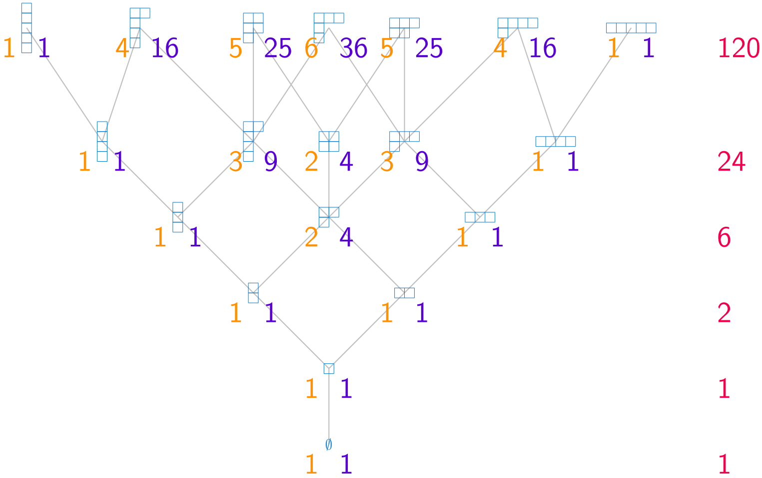
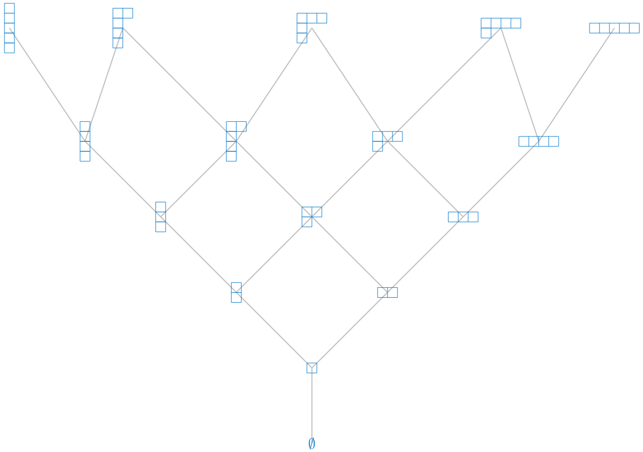


Young's lattice (hook shapes)

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$



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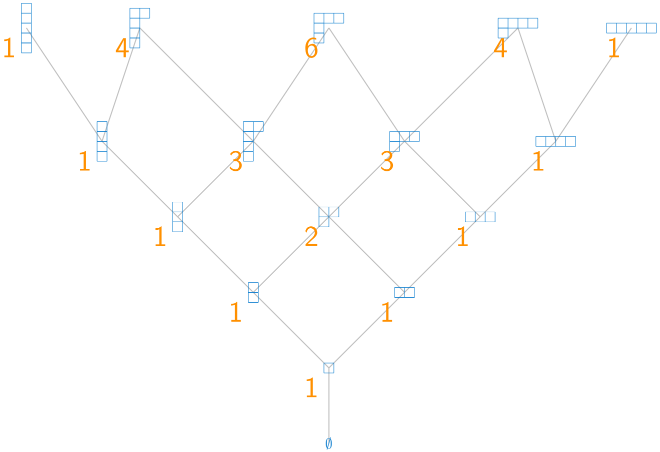

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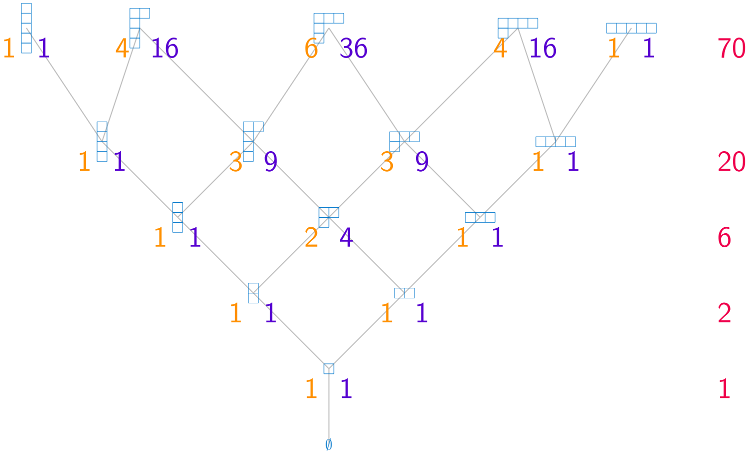

A surprising connection between $n!$ and f_λ

We want a combinatorial proof of

$$\sum_{\lambda \vdash n} f_\lambda^2 = n!.$$

Insertion algorithm

Definition

Given a permutation π, the **insertion tableaux** $P(\pi)$ is obtained by by starting from an empty standard Young tableaux and inserting letters of π from left to right into the first row as follows.

1. If the letter is largest in the row, place it at the end.
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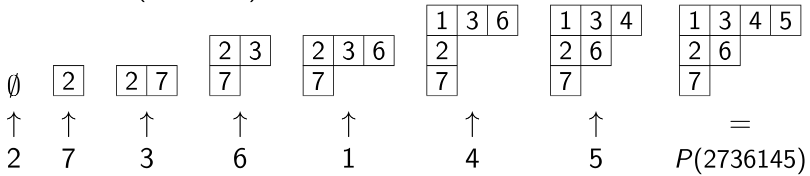
Example

Let's find $P(2736145)$



Recording tableaux

Example

Let's find $P(2736145)$

$$\begin{array}{cccccccc}
\emptyset &
\begin{array}{|c|}\hline 2\\\hline\end{array} &
\begin{array}{|c|c|}\hline 2&7\\\hline\end{array} &
\begin{array}{|c|c|}\hline 2&3\\\hline 7\\\cline{1-1}\end{array} &
\begin{array}{|c|c|c|}\hline 2&3&6\\\hline 7\\\cline{1-1}\end{array} &
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\begin{array}{|c|c|c|c|}\hline 1&3&4&5\\\hline 2&6\\\cline{1-2} 7\\\cline{1-1}\end{array} \\
\uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & = \\
2 & 7 & 3 & 6 & 1 & 4 & 5 & P(2736145)
\end{array}$$

The **recording tableaux** $Q(\pi)$ records the box added to the shape at each step.

$$Q(2736145) = \begin{array}{|c|c|c|c|}\hline 1&2&4&7\\\hline 3&6\\\cline{1-2} 5\\\cline{1-1}\end{array}$$

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The **recording tableaux** $Q(\pi)$ records the box added to the shape at each step.

$$Q(2736145) = \begin{array}{cccc} 1 & 2 & 4 & 7\\ 3 & 6 & & \\ 5 & & & \end{array}$$

With both the insertion and recording tableaux, this process is reversible!

Robinston–Schensted

Theorem (Robinson 1938, Schensted 1961)

The map $\pi \xrightarrow{\mathrm{RS}} (P(\pi), Q(\pi))$ *is a bijection! It takes permutations of* n *to pairs of standard Young tableaux with the same shape* $\lambda \vdash n$.

This is a bijective proof of

$$\sum_{\lambda \vdash n} f_\lambda^2 = n!$$

Question

How do you think $\mathrm{RS}(\pi)$ compares to $\mathrm{RS}(\pi^{-1})$?

A beautiful symmetry

Theorem (Schützenberger)

$$P(\pi) = Q(\pi^{-1})$$
$$Q(\pi) = P(\pi^{-1})$$

Corollary

Involutions *(permutations $\pi = \pi^{-1}$) are in bijection with SYT.*

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Claim

A beautiful symmetry deserves a beautiful explanation.

Fomin growth diagrams

Consider the permutation matrix for our $\pi = 2736145$.

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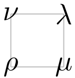
A local map for cell 

1. If there is an X, λ is $\nu = \mu = \rho$ plus a square in the first row.
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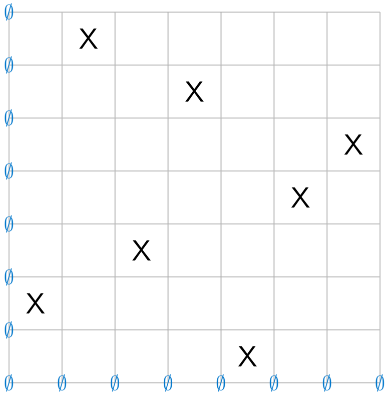
3. Otherwise, $\lambda = \nu \cup \mu$.



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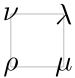
A local map for cell 

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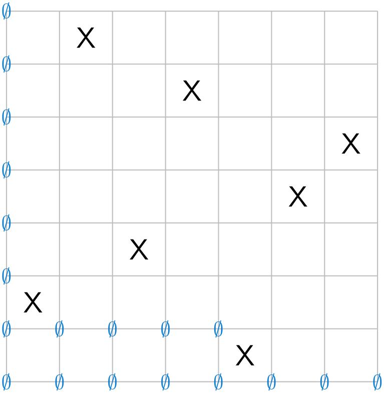

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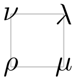
A local map for cell 

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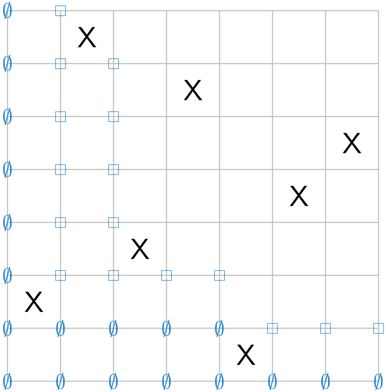

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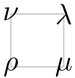
A local map for cell 

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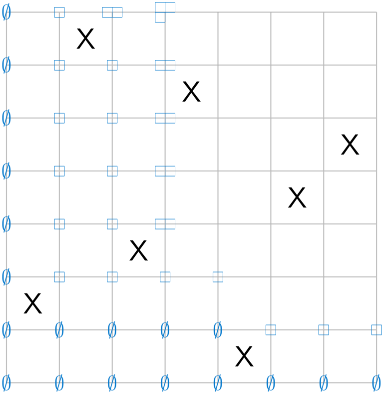

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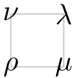
A local map for cell 

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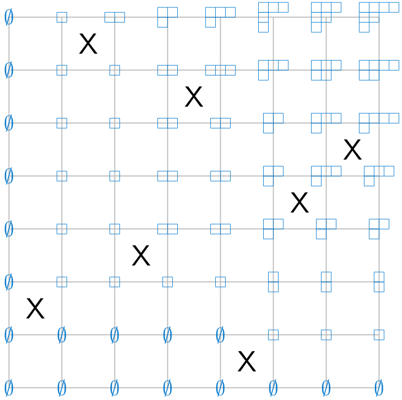

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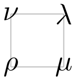
A local map for cell 

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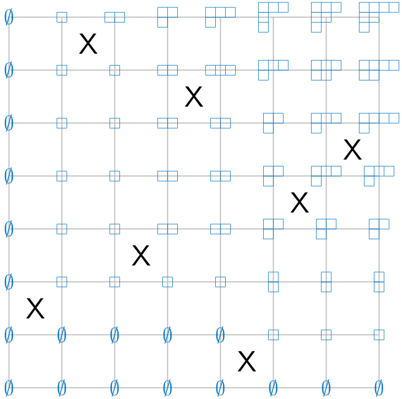

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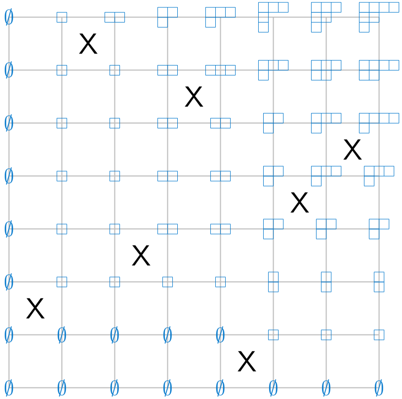
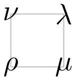


A local map for cell 

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This is reversible! It is another description of Robinson-Schensted.
And it makes $P(\pi) = Q(\pi^{-1})$ clear.

Why?

Question

Why seek bijective proofs?
Why seek beautiful descriptions of bijections?

Why?

Question

Why seek bijective proofs?
Why seek beautiful descriptions of bijections?

They do not simply *verify*, they *clarify*.

Viennot shadow lines

Here's another description. Again consider the permutation matrix for our $\pi = 2736145$.

	X					
			X			
						X
					X	
		X				
X						
				X		

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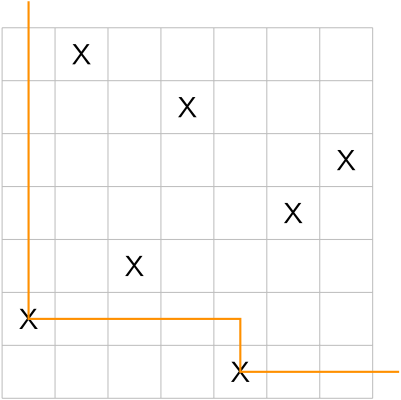

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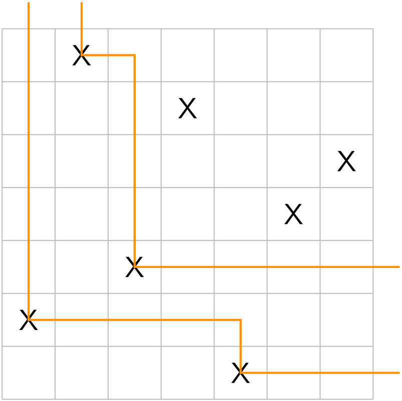

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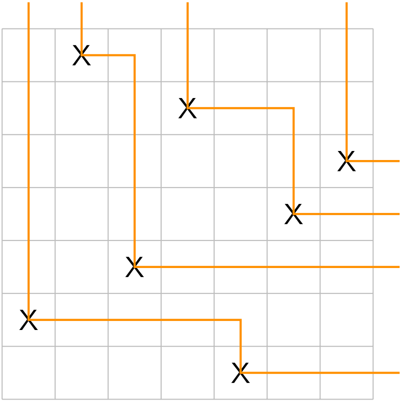

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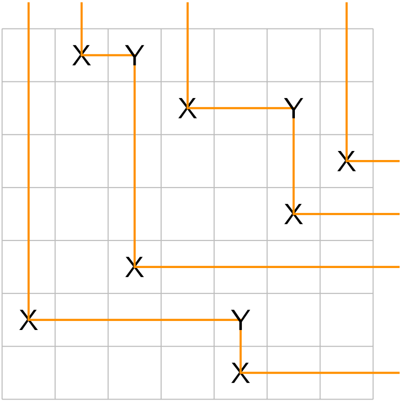

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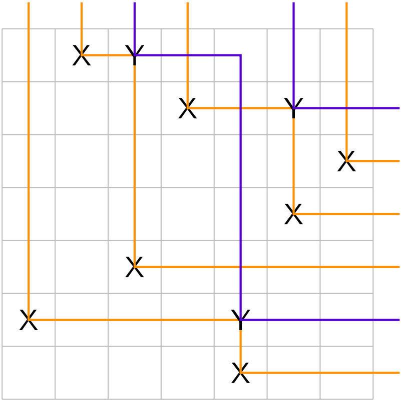

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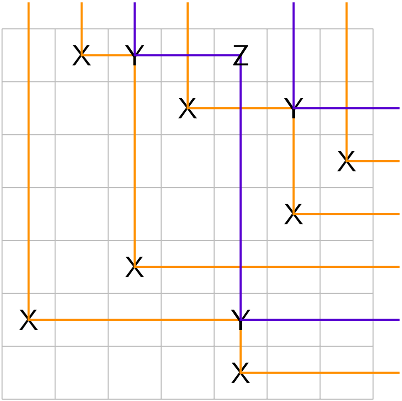

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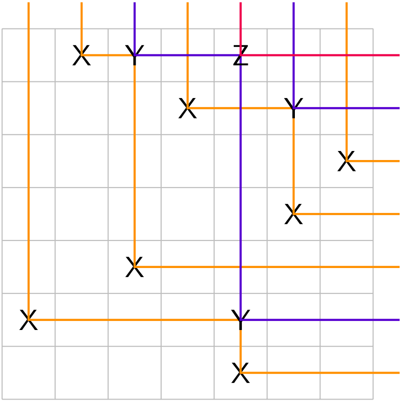

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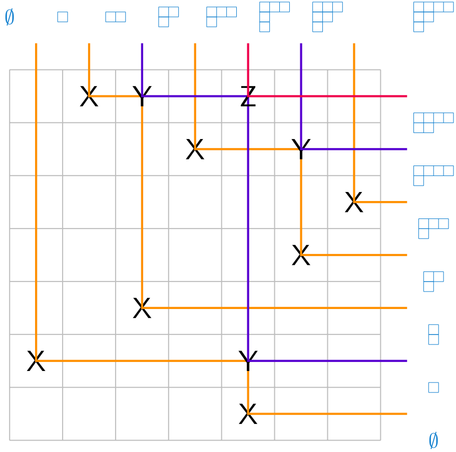

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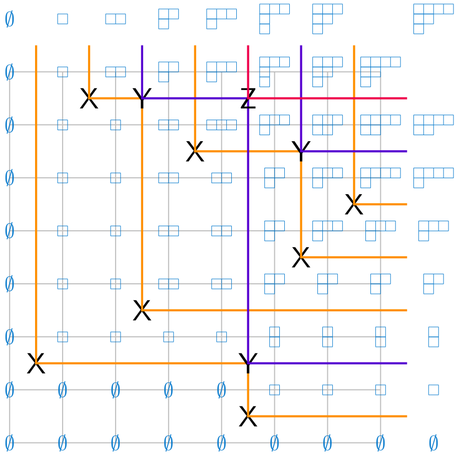

Increasing and decreasing subsequences

The sequence 236 is a subsequence of $\pi = 2736145$. Can you find a larger increasing subsequence?

Increasing and decreasing subsequences

The sequence 236 is a subsequence of $\pi = 2736145$. Can you find a larger increasing subsequence? Yes! 2345 has size 4.

What is the largest union of two increasing subsequences?

Increasing and decreasing subsequences

The sequence 236 is a subsequence of $\pi = 2736145$. Can you find a larger increasing subsequence? Yes! 2345 has size $\mathbf{4}$.

What is the largest union of two increasing subsequences? $236, 145$ has size $\mathbf{6}$. Next, we get $236, 145, 7$ with $\mathbf{7}$.

Increasing and decreasing subsequences

The sequence 236 is a subsequence of $\pi = 2736145$. Can you find a larger increasing subsequence? Yes! 2345 has size **4**.

What is the largest union of two increasing subsequences? $236, 145$ has size **6**. Next, we get $236, 145, 7$ with **7**.

Recall that $P(\pi)$ and $Q(\pi)$ have shape $(4, 2, 1)$, which has partial sums **4,6,7.**

Greene's theorem

Theorem (Greene)

Let $\lambda(\pi) = (\lambda_1, \lambda_2, \dots)$ *be the shape of the tableaux in* $\mathsf{RSK}(\pi)$*. Then* $\lambda_1 + \lambda_2 + \cdots + \lambda_k$ *is the maximal size of a union of* k *increasing subsequences in* π*. Also,* $\lambda_1^\intercal + \lambda_2^\intercal + \cdots + \lambda_k^\intercal$ *is the maximal size of a union of* k *decreasing subsequences in* π*.*

Remark

Combined with the hook-length formula, this allows us to count permutations with given maximal sizes of increasing and decreasing subsequences.

A quick application

Corollary (Erdős-Szekeres 1935, Seidenberg 1959)

If $\pi \in \mathfrak{S}_{nm+1}$, then either π has an increasing subsequence of length n or a decreasing subsequence of length m.

Proof.

Suppose not for some $\pi \in \mathfrak{S}_{nm+1}$ with $\mathrm{RSK}(\pi) = (P, Q)$. Then P and Q must fit inside an $n \times m$ rectangle, and so have at most nm boxes. □

Yet another Robinson–Schensted description

We can use Greene's theorem to give a different description of RS.

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$$\pi = 2736145$$

$\lambda(2)$ $\lambda(27)$ $\lambda(273)$ $\lambda(2736)$ $\lambda(27361)$ $\lambda(273614)$ $\lambda(2736145)$

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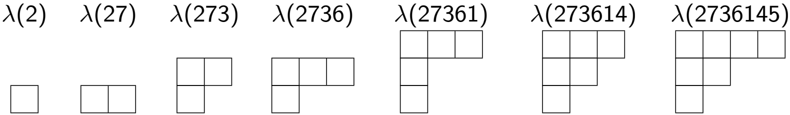

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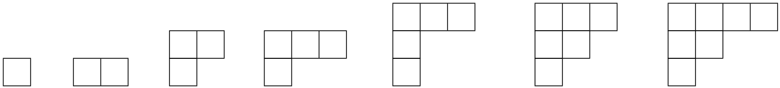
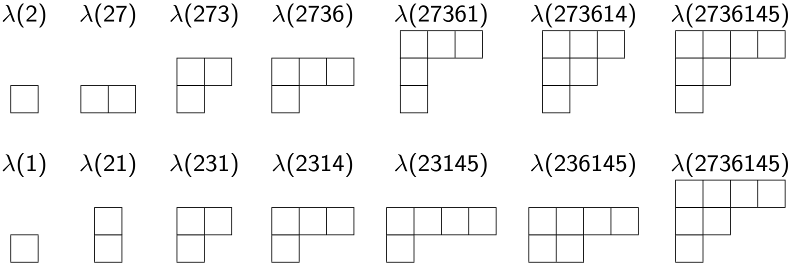


$\lambda(1)$ $\lambda(21)$ $\lambda(231)$ $\lambda(2314)$ $\lambda(23145)$ $\lambda(236145)$ $\lambda(2736145)$

Yet another Robinson–Schensted description

We can use Greene's theorem to give a different description of RS.

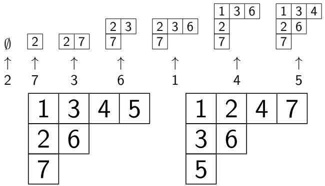
$$\pi = 2736145$$

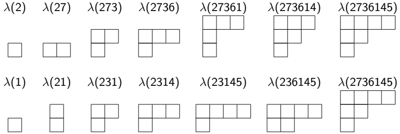


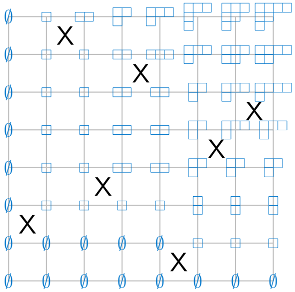
1	3	4	5
2	6		
7			

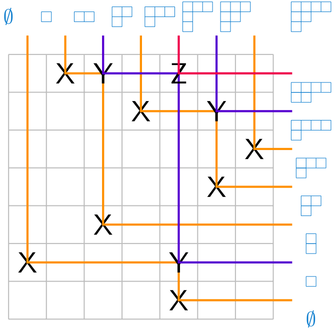
1	2	4	7
3	6		
5			

Four descriptions of Robinson–Schensted









(there are more...)

Semi-standard Young tableaux

Definition

A *semi-standard Young tableau* (SSYT) is weakly increasing on rows and strictly increasing on columns.

1	1	2
2	4	
3	5	
5		

This has weight $x_1^2 x_2^2 x_3 x_4 x_5^2$ in the *Schur function* $s_{(3,2,2,1)}(\mathbf{x})$...

Semi-standard Young tableaux

Semi-standard Young tableaux are enumerated by Schur functions.

Definition

Schur functions are defined by

$$s_\lambda(\mathbf{x}) = \sum_{\mathrm{SSYT}_\lambda} x_1^{\mu_1} x_2^{\mu_2} \cdots$$

where μ_i is the number of times label i appears in the semi-standard Young tableaux.

Schur functions are an important basis for *symmetric functions*. They are *characters* of *irreducible representations* of the *general linear groups*.

Robinson–Schensted–Knuth

Theorem (Knuth)

There is a bijection between $\mathbb{N}$ *matrices and semi-standard Young Tableaux.*

RSK with Fomin growth

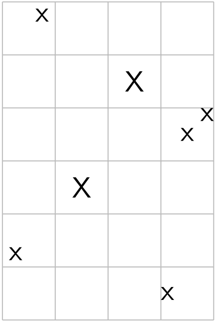
To allow multiples in the same row, column, or cell, simply subdivide and order them left to right and top to bottom.

1			
		1	
			2
	1		
1			
			1

RSK with Fomin growth

To allow multiples in the same row, column, or cell, simply subdivide and order them left to right and top to bottom.

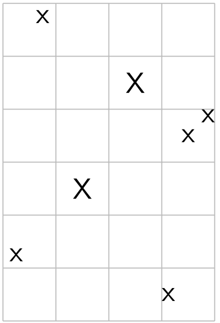
1			
		1	
			2
	1		
1			
			1

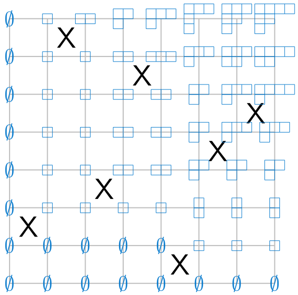


RSK with Fomin growth

To allow multiples in the same row, column, or cell, simply subdivide and order them left to right and top to bottom.

1			
		1	
			2
	1		
1			
			1





1	3	4	5
2	6		
7			

1	2	4	7
3	6		
5			

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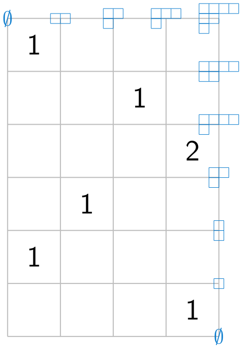
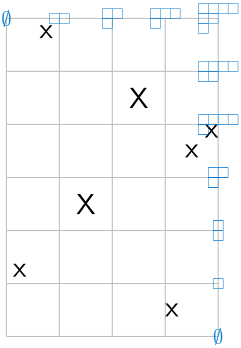
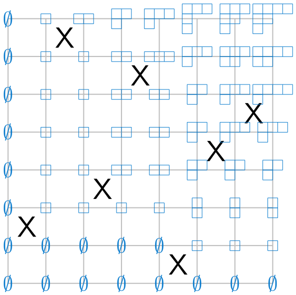


1	3	4	4
2	5		
6			



1	1	3	4
2	4		
4			



1	3	4	5
2	6		
7			

1	2	4	7
3	6		
5			

Variations of RSK

What if we order our X's in another way?

Variations of RSK

What if we order our X's in another way?

1	
1	
	1
1	1

Variations of RSK

What if we order our X's in another way?

		X		
	X			
				X
			X	
X				

RSK

1	
1	
	1
1	1

Variations of RSK

What if we order our X's in another way?

1	
1	
	1
1	1

RSK

		X		
	X			
				X
			X	
X				

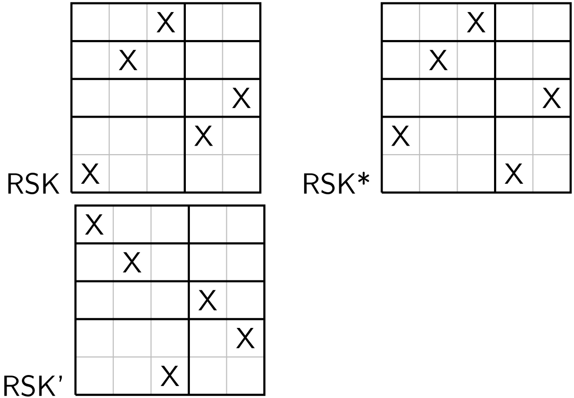
RSK*

		X		
	X			
				X
X				
			X	

Variations of RSK

What if we order our X's in another way?

1	
1	
	1
1	1



Variations of RSK

What if we order our X's in another way?

1	
1	
	1
1	1

RSK

		X		
	X			
				X
			X	
X				

RSK*

		X		
	X			
				X
X				
			X	

RSK'

X				
	X			
			X	
				X
		X		

RSK'*

X				
	X			
			X	
		X		
				X

Cauchy's formulas

Recall the schur polynomials $s_\lambda(\mathbf{x}) = \sum_{\text{SSYT}} x_1^{\mu_1} x_2^{\mu_2} \cdots$ where μ_i is the number of times label i appears in the semi-standard Young tableaux.

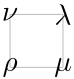
$$\prod_{i,j} \frac{1}{1 - x_i y_j} = \sum_\lambda s_\lambda(\mathbf{x}) s_\lambda(\mathbf{y})$$

$$\prod_{i,j} (1 - x_i y_j) = \sum_\lambda s_{\lambda^\intercal}(\mathbf{x}) s_\lambda(\mathbf{y})$$

RSK gives a combinatorial proof of the first, and RSK* gives a combinatorial proof of the second.

Generalizing Fomin growth

Recall Fomin growth:

A local map for cell 

1. If there is an X, λ is $\nu = \mu = \rho$ plus a square in the first row.
2. If $\nu = \mu \neq \rho$, λ is $\nu = \mu$ plus a square in the row after $\nu \backslash \rho$.
3. Otherwise, $\lambda = \nu \cup \mu$.

This is a local property which takes advantage of the fact that Young's lattice is a *differential poset*.

Differential Posets

Definition

A *differential poset* P

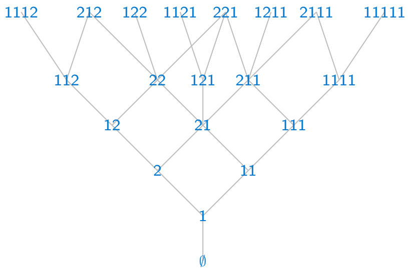
- is locally finite, graded, and has $\hat{0}$
- for every pair, there is either one or zero elements covered by both, and the same number covering both
- every element is covered by one more element than it covers

Example

Young's lattice and the Young-Fibonacci lattice are the most well known.

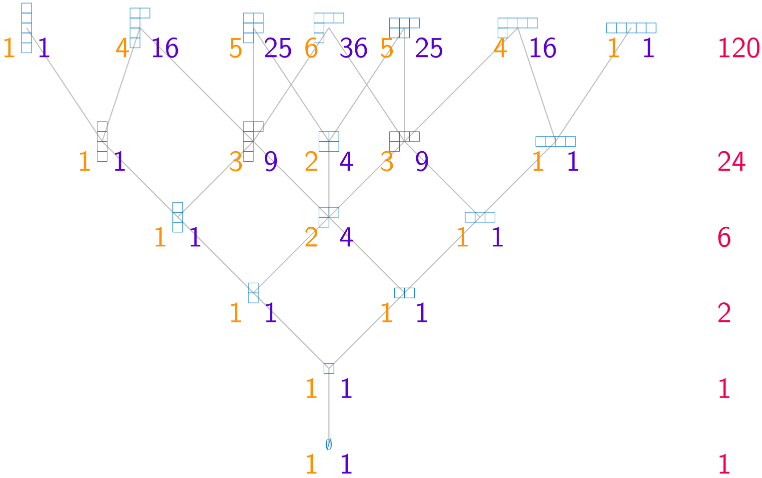
The Young-Fibonacci lattice

- Move up:
  - Insert a 1 to the left of any 1's.
  - Change the leftmost 1 into a 2.
- Move down:
  - Remove leftmost 1.
  - Change a 2 without any 1's to its left to a 1.



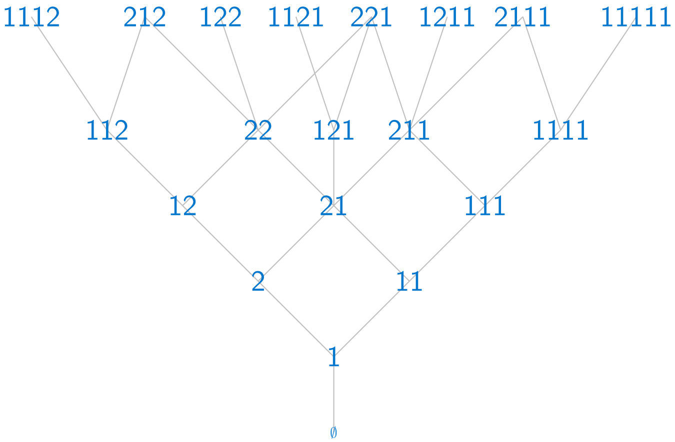
Young's lattice

$$\sum_{\lambda \vdash n} {f_\lambda}^2 = n!$$



Young-Fibonacci lattice

$$\sum_{\lambda \in \mathsf{YFD}(n)} \#\,\mathsf{SYFT}(\lambda)^2 = n!$$



Young-Fibonacci lattice

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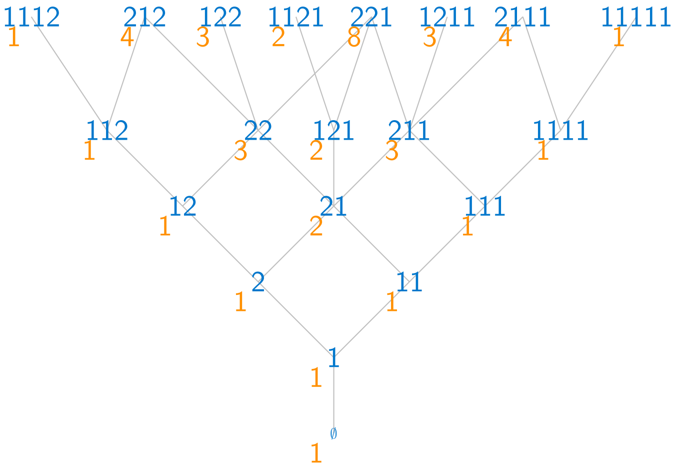

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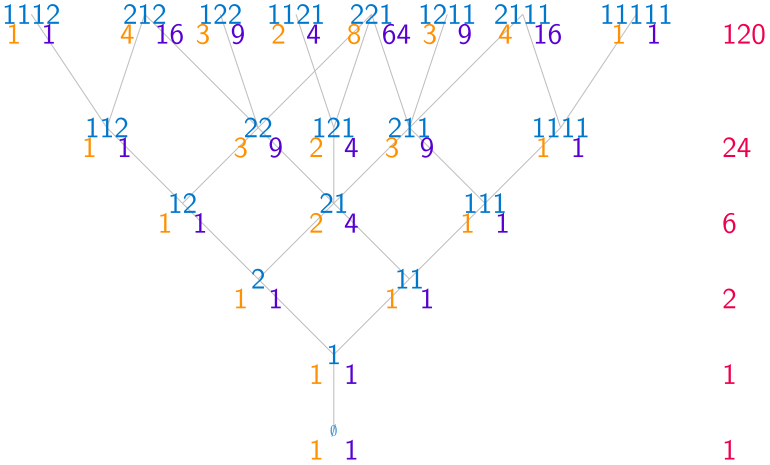

Thank You!

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